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Since Laplace's equation is the equation satisfied by the components of any field of force due to gravitating bodies, this result suggests an explanation of the propagation of gravitation. For we thus see that it is possible to analyse the field of force due to a gravitating body, by a "spectrum analysis" as it were, into an infinite number of constituent fields; and although the whole field of force does not vary with the time, yet each of the constituent fields is of an undulatory character, *i.e.* the disturbance at any point reproduces itself after the lapse of any time at another point whose distance is proportional to the elapsed time. But this result assimilates the propagation of gravitation to that of light; for the undulatory phenomena just described, in which the varying vector is a gravitational force perpendicular to the wave-front, may be compared with those made familiar by the electromagnetic theory of light, in which the varying vectors consist of electric and magnetic forces parallel to the wave-front. The waves are in other respects of the same type in the two cases, and it seems probable that an identical property of the medium ensures their transmission through space. This undulatory theory of gravity would require that gravity should be propagated with a finite velocity, which, however, need not be the same as that of light, and may be enormously greater.

Of course this investigation does not explain the *cause* of gravity; all that is done is to show that the propagation across space of forces which vary as the inverse square of the distance does not require for its explanation any other property of the medium than one which has long been known and accepted.

On the Mean Distance of a Planet, as a Function of Three Heliocentric Distances and the Observed Times. By Shin Hirayama.

§ 1. Let r_1, r_2, r_3 be three heliocentric distances of a planet;
t_1, t_2, t_3 the corresponding observed times;
a the mean distance.

The radius vector of a planet must satisfy the differential equation

$$\frac{d^2s}{dt^2}-\frac{2}{\sqrt{s}}+\frac{2}{a}=0,$$

putting $r^2=s$.

If we let

$$\theta_1=k(t_3-t_2),\ \theta_2=k(t_3-t_1),\ \theta_3=k(t_2-t_1),\ \log k=8{\cdot}235581-10,$$

and express the value r^2 in terms of t by expansion into series, we have

$$s=r^2=a_0+a_1t+a_2t^2+a_3t^3+a_4t^4,$$

where a_0 a_1, a_2, a_3, a_4 are five constants independent of time.

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Now, for $t = -\theta_3$

$$s = s_1, \quad \frac{d^2s}{dt^2} = \frac{2}{\sqrt{s_1}} - \frac{2}{a},$$

for $t = 0$

$$s = s_2, \quad \frac{d^2s_2}{dt^2} = \frac{2}{\sqrt{s_2}} - \frac{2}{a},$$

for $t = \theta_1$

$$s = s_3, \quad \frac{d^2s_3}{dt^2} = \frac{2}{\sqrt{s_3}} - \frac{2}{a}.$$

Hence

$$\begin{aligned} s_1 &= a_0 - \theta_3 a_1 + \theta_3^2 a_2 - \theta_3^3 a_3 + \theta_3^4 a_4, \\ s_2 &= a_0, \\ s_3 &= a_0 + \theta_1 a_1 + \theta_1^2 a_2 + \theta_1^3 a_3 + \theta_1^4 a_4, \\ \frac{2}{\sqrt{s_1}} - \frac{2}{a} &= 2a_2 - 6\theta_3 a_3 + 12\theta_3^2 a_4, \\ \frac{2}{\sqrt{s_2}} - \frac{2}{a} &= 2a_2, \\ \frac{2}{\sqrt{s_3}} - \frac{2}{a} &= 2a_2 + 6\theta_1 a_3 + 12\theta_1^2 a_4. \end{aligned}$$

Eliminating the five constants a_0, a_1, a_2, a_3, a_4 from these six equations, we get a single equation of the form

$$\left.\begin{array}{l} \dfrac{\theta_1\theta_2\theta_3}{a} = 2\left(\dfrac{\theta_1\mu_1}{r_1} + \dfrac{\theta_2\mu_2}{r_2} + \dfrac{\theta_3\mu_3}{r_3}\right) = (\theta_1 r_1^2 - \theta_2 r_2^2 + \theta_3 r_3^2) \\ \text{where} \\ \mu_1 = \dfrac{1}{12}(\theta_2\theta_3 - \theta_1^2), \ \mu_2 = \dfrac{1}{12}(\theta_1\theta_3 + \theta_2^2), \ \mu_3 = \dfrac{1}{12}(\theta_1\theta_2 - \theta_3^2) \end{array}\right\} \quad (1)$$

This is the required equation. A less exact formula is given by Radau in *Bulletin Astronomique*, tome ii. p. 11.

§ 2. For the purpose of testing the above expression (1) I have chosen the *Ceres* example treated by Gauss, Oppolzer, and Gibbs, the interval of extreme observations being about 260 days.

$\log \theta_1 = 0{\cdot}335852$ $\quad \log \mu_1 = 9{\cdot}669211$ $\quad \log r_1 = 0{\cdot}428279$
$\log \theta_2 = 0{\cdot}650362$ $\quad \log \mu_2 = 0{\cdot}318372$ $\quad \log r_2 = 0{\cdot}413281$
$\log \theta_3 = 0{\cdot}362407$ $\quad \log \mu_3 = 9{\cdot}562392$ $\quad \log r_3 = 0{\cdot}406200$

The computation by (1) gives us $\log a = 0{\cdot}442438$,
while Oppolzer gives $\log a = 0{\cdot}442463$.

The coincidence is fairly good, if we consider the long interval of time between two observations.

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§ 3. In the special case of parabolical orbits (1) becomes

$$\theta_1 r_1^2\left(1-\frac{2\mu_1}{r_1^3}\right)-\theta_2 r_2^2\left(1+\frac{2\mu_2}{r_2^3}\right)+\theta_3 r_3^2\left(1-\frac{2\mu_3}{r_3^3}\right)=0 \qquad (2)$$

—a relation between three heliocentric distances and the time.

§ 4. Let E_1, E_2, E_3 be the corresponding excentric anomalies, then we have

$$(r_1-a)\sin(E_3-E_2)-(r_2-a)\sin(E_3-E_1)+(r_3-a)\sin(E_2-E_1)=0,$$

while (1) may be written in the form

$$(r_1-a)\theta_1\left(r_1+a+\frac{2\mu_1}{ar_1}\right)-(r_2-a)\theta_2\left(r_2+a-\frac{2\mu_2}{ar_2}\right)+(r_3-a)\theta_3\left(r_3+a+\frac{2\mu_3}{ar_3}\right)=0.$$

Hence, comparing the two above equations, we see that

$$\sin(E_3-E_2) \qquad \sin(E_3-E_1) \qquad \sin(E_2-E_1)$$

are respectively proportional to

$$\theta_1\left(r_1+a+\frac{2\mu_1}{ar_1}\right) \qquad \theta_2\left(r_2+a-\frac{2\mu_2}{ar_2}\right) \qquad \theta_3\left(r_3+a+\frac{2\mu_3}{ar_3}\right)=0.$$

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, between 1902 *January* 6, *and April* 10.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of *Neptune's* satellite were made from photographs taken with the 26-inch refractor of the Thompson equatorial. The occulting shutter was generally used, and the position of the plate adjusted so that the planet was behind the shutter while the satellite was just outside of it. By means of the shutter, which is pivoted (see *Monthly Notices*, vol. lx. No. 7), short exposures were given at intervals to the planet. Rapid plates were used, and the exposures given to the satellite were from 10^m to 30^m, generally 20^m. For the planet the short exposures, which may be taken as about $\frac{1}{10}^s$ in duration, were given each half-minute. Thus to an exposure of 20^m on the satellite there would be thirty-nine of these short exposures on the planet.

The zero of position-angle was obtained by stopping the clock and giving a short supplementary exposure. Generally, several short exposures were given, the clock being stopped for a short time between each of them, so that each photograph contains, in